

Name: _____ Section Day and Time: _____

Complete the following problems, making sure to SHOW ALL WORK. If you're stuck on something, CLEARLY EXPLAINING what you do know or what you would do will get you partial credit!

1. (a) Simplify the rational expression

$$\frac{x^2 + 3x + 1}{x - 2}$$

by way of polynomial long division.

- (b) Compute

$$\int_0^1 \frac{x^2 + 3x + 1}{x - 2} dx.$$

- (c) Write the definition (in terms of limits and integrals) of the improper integral

$$\int_1^3 \frac{x^2 + 3x + 1}{x - 2} dx.$$

Can you tell quickly whether the integral converges or not?

2. Write, draw, or otherwise create something. Potential points for creativity and how much you help me enjoy grading!

1. (a) We just do some long division to get

$$\begin{array}{r} x + 5 \\ x - 2 \overline{) x^2 + 3x + 1} \\ \underline{-x^2 + 2x} \\ 5x + 1 \\ \underline{-5x + 10} \\ 11 \end{array}$$

This gives us

$$\frac{x^2 + 3x + 1}{x - 2} = x + 5 + \frac{11}{x - 2}.$$

- (b) Using the simplification we found above, we can say

$$\begin{aligned} \int_0^1 \frac{x^2 + 3x + 1}{x - 2} dx &= \int_0^1 x + 5 + \frac{11}{x - 2} dx \\ &= \left. \frac{1}{2}x^2 + 5x + 11 \ln|x - 2| \right|_0^1 \\ &= \frac{1}{2} + 5 + 0 - 0 - 0 - 11 \ln|-2| \\ &= \frac{11}{2} - 11 \ln(2). \end{aligned}$$

(c) By definition, since our function is discontinuous at $x = 2$, we have

$$\begin{aligned}\int_1^3 \frac{x^2 + 3x + 1}{x - 2} dx &= \lim_{t \rightarrow 2^-} \int_1^t \frac{x^2 + 3x + 1}{x - 2} dx + \lim_{s \rightarrow 2^+} \int_s^3 \frac{x^2 + 3x + 1}{x - 2} dx \\ &= \lim_{t \rightarrow 2^-} \left. \frac{1}{2}x^2 + 5x + 11 \ln |x - 2| \right|_1^t + \lim_{s \rightarrow 2^+} \left. \frac{1}{2}x^2 + 5x + 11 \ln |x - 2| \right|_s^3.\end{aligned}$$

We actually can tell that this integral *diverges* because, just looking at the first half, we are going to find

$$\lim_{t \rightarrow 2^-} \ln |t - 2| = -\infty.$$

2. Good luck on your finals! I hope you enjoyed this class and our sections at least a little bit! ☺